Coherent Radiation in Gamma-Ray Bursts and Relativistic Collisionless Shocks

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We suggest that coherent radiation may occur in relativistic collisionless shocks via two-stream Weibel instabilities. The coherence amplifies the radiation power by many orders [$\sim 10^{12}$ in Gamma-Ray Bursts (GRBs)] and particles cool very fast before being randomized. We imply (1) GRBs accompany strong infrared emission, (2) protons efficiently transfer energy to electrons and (3) prompt GRBs might be the upscattered coherent radiation.

§1. Introduction

Gamma-Ray Bursts (GRBs) are believed to be produced by the dissipation of the kinetic energy of a relativistic outflow, energized by a central compact object, via collisionless shocks.[1] Although the leading GRB mechanism is synchrotron emission from accelerated electrons, many problems remain unsolved and the true mechanism is far from understood.[2] In view of recent observational discoveries, such as the X-ray flashes,[3–5] the prompt infrared-optical emission,[6,7] and the empirical relations of the peak energy of GRBs that may enable precision cosmology,[8–10] it is high time to reveal the GRB emission mechanism, hopefully from the first principle. One of the most promising approaches is to investigate the kinematics of the relativistic collisionless shocks.[11–13]

In relativistic collisionless shocks, plasma particles are initially interpenetrating. The velocity distribution is anisotropic and the anisotropy drives the Weibel two-stream instability.[11,14–16] The intersecting particles are deflected by a magnetic field perturbation so that the currents making the magnetic fields increase. As a result the magnetic fields perpendicular to the shock propagation are amplified and many current filaments (cylindrical beams) parallel to the shock propagation are generated.[17] Each beam carries a current producing a magnetic field around itself. Since like currents attract each other, they merge and grow in size.[12,18,19] The magnetic fields also grow and reach the maximum when the current growth saturates at the Alfvén limiting current[13] and the particles are randomized.

In this Letter we will add one more key to the above picture by suggesting that current filaments could emit coherent radiation before particles are randomized. A filament emits radiation because charged particles in the filament have a common acceleration when the filament is curved in order to merge. Many particles accelerate in the same way, so that radiation from N particles are added coherently (i.e., $E \sim NE_1$) and the radiation power ($\propto |E|^2$) becomes $\propto N^2$ rather than $\propto N$.[*] The radiation power is amplified by many orders ($\sim 10^{12}$ in GRBs) and particles may lose almost all energy within the turnaround radius. This coherent radiation may have

[*] Our mechanism of coherent radiation is different from that of Ref. 20).

significant impacts on the GRB model: (1) the GRBs may be associated with strong infrared emission, (2) the proton energy may be efficiently transfered to electrons, and (3) even the GRB itself might be the upscattered coherent radiation.

§2. Coherent radiation in relativistic collisionless shocks

To model a single current filament, we consider particles with charge q and mass m running in a filament with a velocity $c\beta = c(1-\gamma^{-2})^{1/2} \sim c[1-(1/2\gamma^2)]$. The filament has a radius λ and is bent with a curvature radius $r > \lambda$. Although this is a simple model, we can quantify the plausibility of the coherent radiation. All quantities are measured in the shocked frame.

According to the linear theory of the Weibel instability,[11),21)] the filament radius is about the order of the plasma skin depth*)

$$\lambda \sim \left(\frac{\pi m c^2}{q^2 n}\right)^{1/2} \sim 3.3 n_{12}^{-1/2} (m/m_e)^{1/2} \text{ cm}, \tag{2·1}$$

where m_e is the electron mass, and we adopt the particle number density $n = 10^{12} n_{12}$ cm^{-3} as typical parameters for the internal shocks of the GRBs. (Note that the possibility of coherent radiation does not depend on the density n. See below.)

The typical curvature radius r of the filament would be about or larger than the Larmor radius

$$r_L = \frac{\gamma m c^2}{qB} \sim 12 \epsilon_{B,-2}^{-1/2} n_{12}^{-1/2} \gamma_1^{1/2} (m/m_e)^{1/2} \text{ cm}, \tag{2·2}$$

where $\gamma = 10\gamma_1$ is a typical relative Lorentz factor in the GRB internal shocks**) and $\epsilon_B = B^2/8\pi n \gamma m c^2 = 10^{-2}\epsilon_{B,-2}$ is a fraction of the magnetic energy. The curvature radius r could be larger than r_L, for example, in the case of a proton filament Debye-shielded by electrons. [See Fig. 1 of Ref. 22) and Fig. 2 of Ref. 18).] So we leave the curvature radius r as a free parameter and consider two cases:

$$(A)\ r > \gamma^2\lambda, \quad (B)\ \gamma^2\lambda \geq r > \lambda. \tag{2·3}$$

Note that $r_L \sim \gamma^{1/2}\lambda$ and hence $r \geq r_L > \lambda$ for relativistic shocks $\gamma \gg 1$ regardless of the density n and the particle mass m from equations (2·1) and (2·2).

Given the curvature radius r of the filament, we can estimate the radiation power from a single particle $P = 2cq^2\beta^4\gamma^4/3r^2$ and thereby the cooling length (without a coherent effect)

$$l_1 \sim \frac{\gamma m c^3}{P} \sim \frac{3r^2 m c^2}{2q^2\beta^4\gamma^3} \sim 5.3 \times 10^{13} r_2^2 \gamma_1^{-3} (m/m_e) \text{ cm}, \tag{2·4}$$

where $r = 10^2 r_2$ cm. The cooling length l_1 is much larger than the other scales, r and λ, for typical parameters. However, if N particles radiate coherently, the cooling

*) More precisely the filament radius depends also on the velocity distribution perpendicular to the propagation.[11),21)] The radius also increases when current filaments merge.

**) Before the internal shocks each ejecta would be cold because of the adiabatic expansion.

length could be N times shorter ($l_N \sim l_1/N$) because the mass m and charge q in equation (2·4) $l_1 \propto m/q^2$ are replaced by Nm and Nq. Coherent radiation has been also discussed in the context of pulsars[23] and particle accelerators.[24),25]

2.1. *The case (A) $r > \gamma^2\lambda$*

In order to estimate the possible number of particles that can radiate coherently N, we remember that a relativistic particle emits radiation ahead into a cone of an angular size[26]

$$\Delta\theta \sim \gamma^{-1}. \tag{2·5}$$

Because of this relativistic beaming, an observer will see radiation from the particle's path length of $\Delta s \sim r\Delta\theta \sim r/\gamma$. After passing Δs the radiation extends to $\sim r(\Delta\theta)^2 \sim r/\gamma^2$ in the perpendicular direction to the propagation. Since this ($\sim r/\gamma^2$) is larger than the filament radius $\lambda < r/\gamma^2$ in the case (A), radiation from the inside of the filament may be coherently superimposed. In the propagation direction, the front and back of the radiation is separated by $\sim r(\Delta\theta)^3 \sim r/\gamma^3$ after passing $\Delta s \sim r\Delta\theta$. Then the characteristic frequency of the radiation is

$$\nu_* \sim \frac{c}{r(\Delta\theta)^3} \sim \frac{c\gamma^3}{r} \sim 3.0 \times 10^{11} r_2^{-1}\gamma_1^3 \text{ Hz}, \tag{2·6}$$

and particles within the wave length $\sim r(\Delta\theta)^3 \sim r/\gamma^3$ may be coherent. Therefore the possible number of coherent particles is given by

$$N \sim n(\pi\lambda^2)[r(\Delta\theta)^3] \sim 3.5 \times 10^{12} r_2 \gamma_1^{-3}(m/m_e), \tag{2·7}$$

and the cooling length may be as short as

$$\frac{l_N}{r} \sim \frac{l_1/N}{r} \sim \frac{3}{2\pi^2\beta^4} \sim 0.15. \tag{2·8}$$

Surprisingly particles may emit almost all energy within a very short distance $\sim 0.1r \sim 10r_2$ cm before turning π radian! An interesting point is that the ratio l_N/r is just a constant and does not depend on other parameters if $\gamma \gg 1$.

2.2. *The case (B) $\gamma^2\lambda \geq r > \lambda$*

If the frequency of interest is the characteristic frequency $\nu_* \sim c\gamma^3/r$, not all particles within the filament radius can be coherent since the perpendicular extent of the radiation $\sim r(\Delta\theta)^2 \sim r/\gamma^2$ is smaller than the filament radius $\lambda > r/\gamma^2$. However, if the frequency of interest ν is less than ν_*, the radiation is beamed into a wider angular size

$$\Delta\theta \sim \gamma^{-1}(\nu/\nu_*)^{-1/3}, \tag{2·9}$$

than $\sim \gamma^{-1}$. Then, at a frequency lower than

$$\nu \sim \nu_* \left(\frac{r}{\gamma^2\lambda}\right)^{3/2}, \tag{2·10}$$

the radiation from the inside of the filament may be coherent since $r(\Delta\theta)^2 > \lambda$. The wave length of the radiation is $\sim r(\Delta\theta)^3 \sim r\gamma^{-3}(\nu/\nu_*)^{-1}$. [Note that this is less than the plasma skin depth λ at a frequency in equation (2·10).] Therefore the possible number of coherent particles $N \sim n(\pi\lambda^2)[r(\Delta\theta)^3]$ is larger than that in equation (2·7) by $\sim (\nu/\nu_*)^{-1}$. On the other hand the incoherent cooling length l_1 should be multiplied by $\sim (\nu/\nu_*)^{-4/3}$ because the radiation power from a single particle at a frequency ν is $\sim (\nu/\nu_*)^{4/3}$ times smaller than that at ν_*. Consequently, replacing l_1 and N by $l_1(\nu/\nu_*)^{-4/3}$ and $N(\nu/\nu_*)^{-1}$ in equation (2·8), we obtain

$$\frac{l_N}{r} \sim 0.15 \left(\frac{\nu}{\nu_*}\right)^{-1/3} \sim 0.15 \left(\frac{\gamma^2\lambda}{r}\right)^{1/2}, \tag{2·11}$$

where we use equation (2·10) in the last equality. Since the curvature radius is larger than the Larmor radius $r > r_L \sim \gamma^{1/2}\lambda$, we have $l_N/r < 0.1\gamma^{3/4}$. Therefore particles can lose their energy within r as long as $\gamma \lesssim 20$ in the case (B). Note that coherent radiation in the case (B) is less efficient than that in the case (A).

2.3. *Radiative instability*

In order for the radiation to be coherent, the filament needs to be clumpy with the scale of the wave length $\sim r(\Delta\theta)^3$. Otherwise the particle distribution in the filament is random and the emission is added incoherently. Since the Weibel instability only makes structures perpendicular to the propagation, other mechanisms are necessary to bunch the filament.

A clumpy filament may be produced spontaneously by the radiative instability.[23] A density perturbation generates some coherent radiation and the interaction of this radiation with particles may amplify the density perturbation.

Goldreich and Keeley[23] studied the stability of a ring of monoenergetic relativistic particles and applied to pulsars. They found the unstable condition as

$$\left[\gamma^3(\Delta\theta)^3\right]^{4/3} \left(\frac{\gamma^3}{\pi\lambda^2 rn}\right)\left(\frac{\gamma^2 q^2}{rmc^2}\right) < 1, \tag{2·12}$$

which gives $8 \times 10^{-26}\gamma_1^5 r_2^{-2}(\nu/\nu_*)^{-4/3}(m/m_e)^{-2} < 1$ and is well satisfied in a large parameter space. They also derived the growth length d as

$$\frac{d}{r} \sim \left(\frac{rmc^2}{q^2}\frac{\gamma^3}{\pi\lambda^2 rn}\right)^{1/2}(\Delta\theta)^2 \sim \frac{1}{\pi\gamma^{1/2}}\left(\frac{\nu}{\nu_*}\right)^{-2/3}. \tag{2·13}$$

In the case (A) we have $\nu \sim \nu_*$ and so $d/r \sim 1/\pi\gamma^{1/2} < 1$. Therefore the density fluctuation could be amplified within the curvature radius r for the coherent radiation to be efficient. In the case (B) we have $d/r \sim \gamma^{3/2}\lambda/\pi r < \gamma^{1/2}/\pi$ where we use equation (2·10) and $r > r_L \sim \gamma^{1/2}\lambda$. So the density fluctuation may grow depending on the values of r and γ.

§3. Possible GRB scenarios

In the previous section we suggest a possibility that particles could lose almost all energy in the relativistic collisionless shocks very fast by emitting coherent radiation.

Let us consider implications of this possibility for the GRB models.

The first possibility is that strong infrared emission may be associated with GRBs. The Weibel instability grows faster for electrons than for protons, and electrons may emit almost all energy by coherent radiation. Then the radiation may carry $m_e/m_p \sim 10^{-3}$ of all kinetic energy, and has a frequency about

$$\nu_{*,\rm obs} \sim \Gamma \nu_* \sim 3 \times 10^{13} r_2^{-1} \gamma_1^3 \Gamma_2 \ {\rm Hz}, \tag{3·1}$$

where ν_* is given by equation (2·6) and $\Gamma = 10^2 \Gamma_2$ is the Lorentz factor of the shocked ejecta. The coherent radiation may stand out above the prompt GRB, which has only $\sim \nu_{*,\rm obs}/100{\rm keV} \sim 10^{-6}$ of all kinetic energy at the infrared $\nu_{*,\rm obs}$.[*)]

Second the coherent radiation could efficiently transfer energy from protons to electrons. Before being shocked protons carry $m_p/m_e \sim 10^3$ times more energy than electrons. In the standard GRB model[1)] we assume that a large fraction of the proton energy is converted to the electron energy to explain the GRB spectra by the synchrotron radiation of electrons, but no promising mechanism of the energy transfer has been proposed so far.[28)] If protons emit coherent radiation, its frequency is about $\sim 3 \times 10^{11} r_4^{-1} \gamma_1^3 \Gamma_2$ Hz, which is less than the synchrotron self-absorption frequency[27)] and also than the plasma frequency of cold electrons. Therefore the radiation energy would be absorbed by electrons efficiently and electrons would be heated up to the Lorentz factor of $\gamma_e \sim (m_p/m_e)\gamma \sim 10^4 \gamma_1$. Then the standard GRB model may be realized after heated electrons are accelerated. Note that the electron heating should be relevant to the injection problem of the acceleration.

The heated electrons may emit prompt GRBs by the inverse Compton radiation, not by the synchrotron radiation. Interestingly, if the coherent radiation from electrons is upscattered, the typical energy is $\sim \gamma_e^2 \nu_{*,\rm obs} \sim$ MeV, comparable to the peak energy of the GRBs. In this scenario it is not necessary for the magnetic field to survive for a long time.[12),13)]

§4. Discussions

Above calculations are just order-of-magnitude estimates. To establish the possibility of coherent radiation in relativistic collisionless shocks, three-dimensional kinetic simulations are necessary. In principle particle-in-cell simulations[29)–31)] automatically include the coherent effect, though these are challenging. To resolve the wave length $\sim r/\gamma^3$ with at least ~ 10 grids and cover the curvature radius $\sim r$, we need about $\sim [1000(\gamma/5)^3]^3$ grids.

We have considered only the prompt GRBs as a first step. The coherent radiation may be also important for the GRB afterglows, in which the energy transfer from protons to electrons may depend on the Lorentz factor γ as implied by equation (2·11). Applications to other astrophysical phenomena, such as AGN jets, microquasars, giant flares from soft gamma-ray repeaters,[32)] and pulsar wind nebulae are also interesting.

[*)] Note that the synchrotron self-absorption seems to be absent at least in the prompt infrared-optical emission of GRB 041219a.[6),7),27)]

Our simple model of a current filament does not include several effects. We have neglected the velocity distribution parallel and perpendicular to the propagation. The filament radius may be varying. Other bunching mechanisms such as the sausage instability may be more important than the radiative instability. Also the Razin effect may be important since the frequency of coherent radiation is relatively low. When we calculate the upscattered coherent radiation, we may have to take the anisotropy of the radiation field into account.[33] These are interesting future problems.

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